TOPIC 14: FOUL OR NOT?

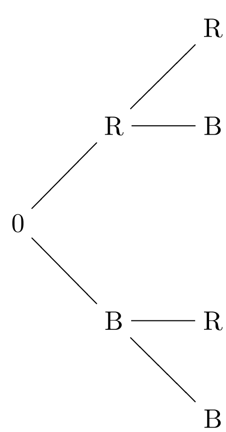
Strategy is certainly crucial to any sport or any situation where we have a confrontation or conflict. In this section, we will see how probability and expected value can help us make better decisions such as whether to foul or not in order to keep our lead at the end of a basketball game or whether to run the ball or pass it in football or even whether the goalie should dive to the left or to the right on a soccer penalty.

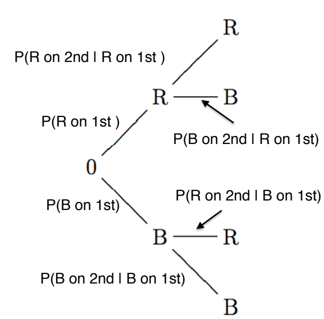
1. Tree Diagrams

Sometimes, if there are sequential steps in an experiment, or repeated trials of the same experiment, or if there are a number of stages of classification for objects sampled, it is very useful to represent the probability/information on a tree diagram.

Example 1.1. *Given an Urn containing 6 red marbles and 4 blue marbles, I draw a marble at random from the urn and then, without replacing the first marble, I draw a second marble from the urn. What is the probability that both marbles are red?*

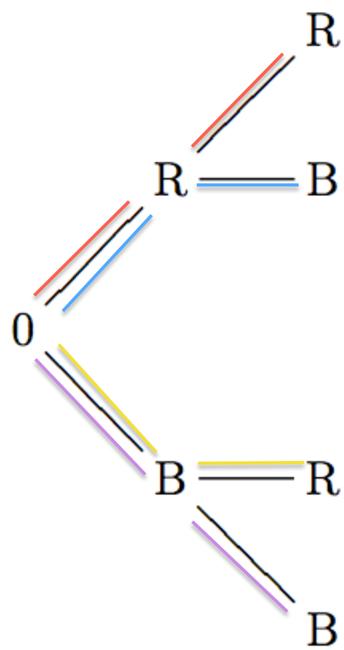
We can draw a tree diagram to represent the possible outcomes of the above experiment and label it with the appropriate conditional probabilities as shown (where 1st denotes the first draw and 2nd denotes the second draw):





(a) Fill in the appropriate probabilities on the tree diagram on the left above (note: the "chemistry" in the urn changes when you do not replace the first ball drawn).

Note that each path on the tree diagram represents one outcome in the sample space.



Outcome	Probability
RR(red path)	
RB(blue path)	
BR(yellow path)	
BB(purple path)	

To find the probability of an outcome we multiply probabilities along the paths;
$P(RR) = P(R \text{ on 1st})P(R \text{ on 2nd}|R \text{ on 1st})$ etc....
(Here we are using the formula $P(A \cap B) = P(A)P(B|A)$.)

(b) Fill in the probabilities for the 4 outcomes in our sample space in the table on the right. Note that this is not an equally likely sample space.

To find the probability of an event, we identify the outcomes (paths) in that event and add their probabilities.

(c) What is the probability that both marbles are red?

(d) What is the probability that the second marble is blue? Note that there are two paths in this event.

Summary of "rules" for drawing tree diagrams

(a) The branches emanating from each point (that is branches on the immediate right) must represent all possible outcomes in the next stage of classification or in the next experiment.

(b) The sum of the probabilities on this bunch of branches adds to 1

(c) We label the paths with appropriate conditional probabilities.

Rules for calculation

(1) Each path corresponds to some outcome

(2) The probability of that outcome is the product of the probabilities along the path

(3) To calculate the probability of an event E, collect all paths in the event E, calculate the probability for each such path and then add the probabilities of those paths.

1.1. **Choosing a strategy with a tree diagram.** Tree diagrams are very helpful when choosing strategies which maximize the probability of winning or expected gains. In some of these situations, the only opponent is chance itself and in others we have an opponent with players making alternate decisions as in chess (we will consider the case where players make simultaneous decisions in the next section).

1.2. **Endgame Basketball.** In basketball the strategy of fouling in the situation where there is very little time left on the clock, the opponent has possession of the ball and your team has a three point lead is a controversial one. To decide on the best strategy one must consider all possible scenarios and the probability that your opponent will win in each. We will use the probabilities given in the following paper by Bill Fenlon ,head coach at DePaw University; `Up Three, To Foul or Not to Foul.` A number of other articles on the topic are also available, for an overview see this article: `college-basketball-strategy-review.`

We assume that the opponent will attempt a three point shot. If If you foul your opponent, the opponent will be granted a number of free throws (F.T.):

- If you foul as the opponent is shooting and the shot is made, your opponent will be granted one free throw.
- If you foul as the opponent is shooting and the shot is missed, your opponent will be granted three free throws.
- If you foul before your opponent shoots, your opponent will be granted two free throws.

Each free throw made is worth one point. If the teams draw, the game will go into overtime(O.T.).

Estimates of Probability You may wish to replace some of the the following probabilities estimated by Coach Fenlon with conditional probabilities based on the strength of your defense, the strength of the opposing the team you are playing or the league in which you are playing. A win for your team is denoted by W and a loss for your team is denoted by L.

(a) Coach Fenlon estimates that the probability that the other team will make a free throw (if that is their intention) in this situation is 0.67.
(b) He estimates that with practice, you can reduce the probability of fouling while the opponent is shooting to 0.02.
(c) He points out that if you foul before the opponent takes the 3 point shot and they are granted 2 free throws, they will attempt to miss the second free throw and go for a rebound shot. In this case, he estimates that they will accidentally make the second shot 2% of the time.
(d) If you do not foul, he estimates that the opposing team will get a chance to make a 3 point shot 5% of the time and if they attempt the shot, they will be successful 20% of the time.
(e) Coach Fenlon also gives other detailed estimates which we have represented on a less detailed tree diagram on the next page.

(i) Using the tree diagram (attached at end of notes), fill in the following probability distribution for the results at the end of regulation time (normal duration of the game) for both strategies:

Strategy Foul

Outcome	Probability
Win (W)	
Loss(L)	
Overtime(O.T.)	

Strategy: Do Not Foul

Outcome	Probability
Win (W)	
Loss(L)	
Overtime(O.T.)	

(ii) Coach Fenton does not give a probability that the opposing team will win in overtime, but does say that they will have the advantage of the momentum of the success in the final seconds. Suppose the probability that your opponent will win in overtime if the game goes into overtime is 0.6 (and the probability that you will win is 0.4). What is the probability that your team will win the game (after final score) for both strategies:

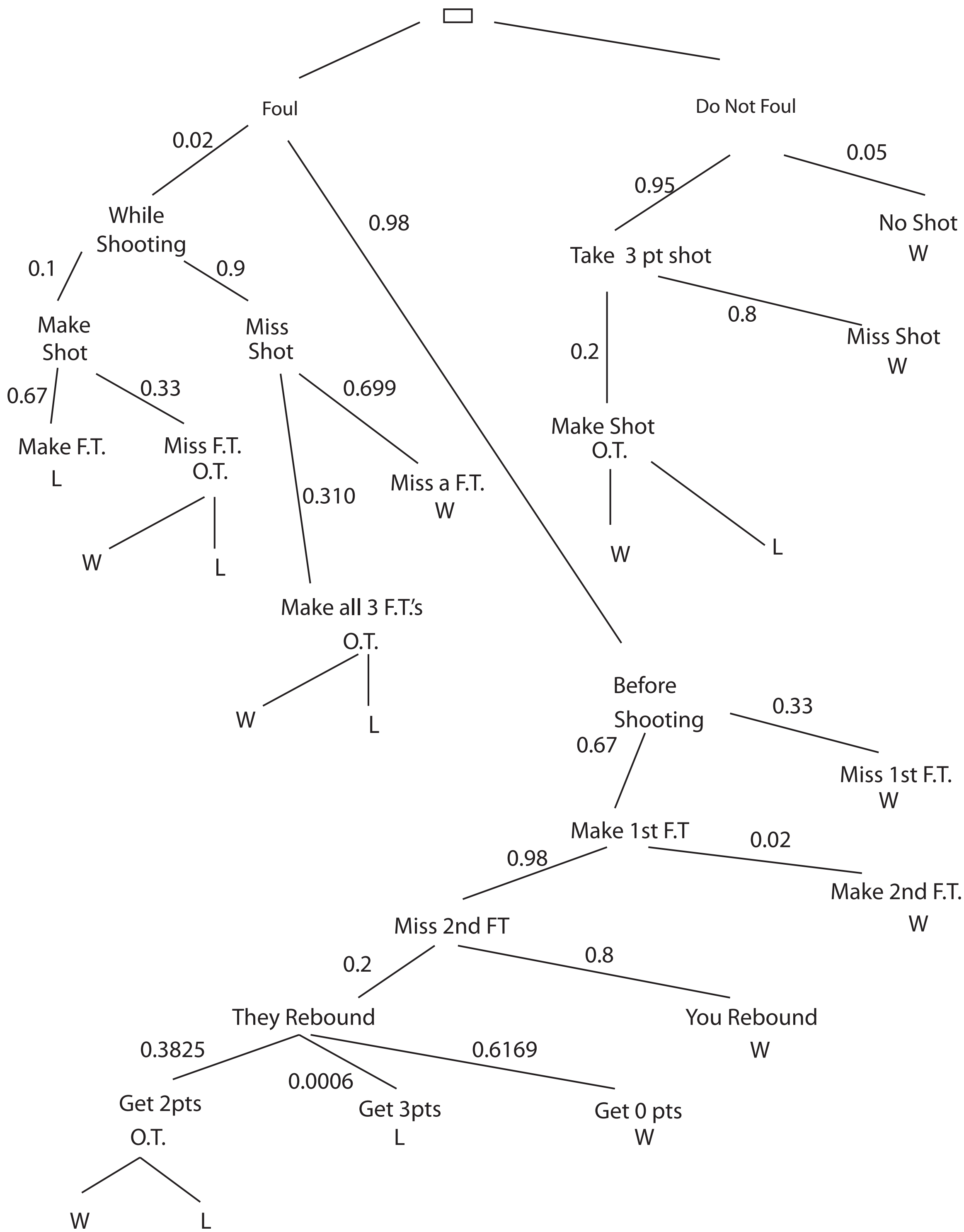
Strategy Foul

Outcome	Probability
Win (W)	
Loss(L)	

Strategy: Do Not Foul

Outcome	Probability
Win (W)	
Loss(L)	

(iii) Your team should choose the strategy which maximizes the probability that they will win. Using the above probabilities, decide on the best strategy for the defensive team in this endgame scenario , "Foul" or "Do not Foul".

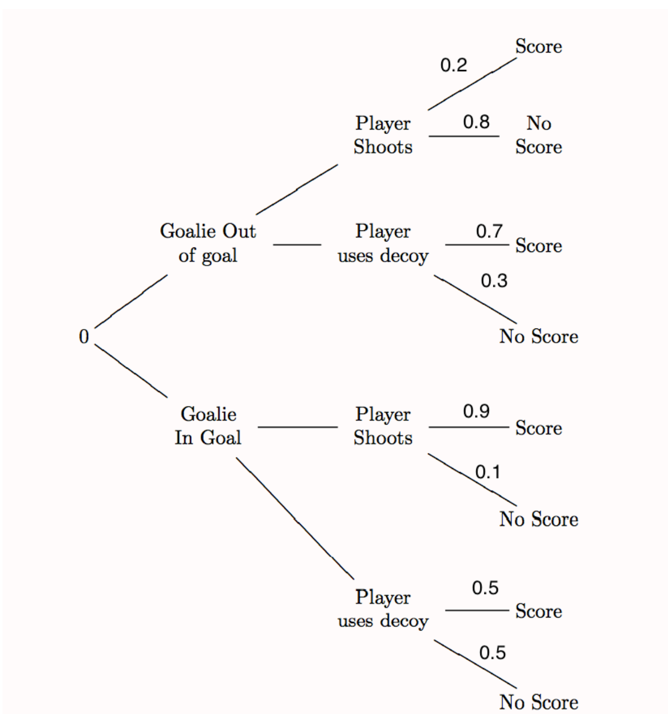

Foul
Do Not Foul
0.02
0.98
0.95
0.05
While Shooting
Take 3 pt shot
No Shot
W
0.1
0.9
0.8
0.2
Make Shot
Miss Shot
Miss Shot
W
0.67
0.33
0.699
0.310
Make F.T.
L
Miss F.T.
O.T.
Miss a F.T.
W
Make Shot
O.T.
W
L
W
L
Make all 3 F.T.'s
O.T.
W
L
Before Shooting
0.33
0.67
Miss 1st F.T.
W
Make 1st F.T
0.98
0.02
Make 2nd F.T.
W
Miss 2nd FT
0.2
0.8
They Rebound
You Rebound
W
0.3825
0.0006
0.6169
Get 2pts
O.T.
Get 3pts
L
Get 0 pts
W
W
L

1.3. **Alternate Move Games.** Sometimes in a game, a player gets to see their opponent's strategy before deciding on their strategy and sometimes both decide on a strategy simultaneously. If the opponent is predictable or telegraphs his/her moves, then we can assume that the players are playing an Alternate move game and a tree diagram should help to choose a strategy.

Example 1.2. *If a hockey player is about to take a shot on goal, the strategy he will use depends on the position of the goalie. If the goalie is out from the goal he may have to use a decoy to get around the goalie, but if the goalie is in the goal (or deep in the crease) then shooting is more likely to be a better strategy than using a decoy.*

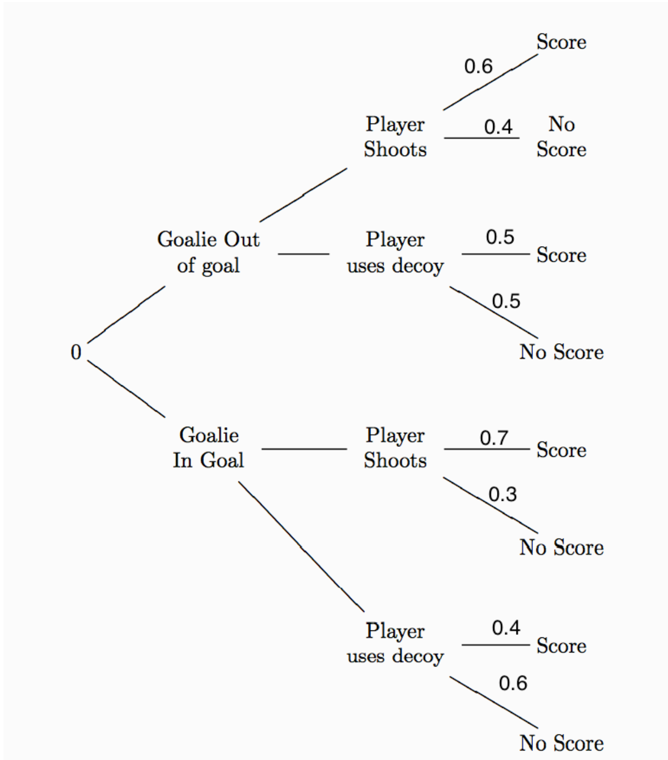
Of course, there are gray areas in between and many moves to choose from and much depends on the position of the other players. However, the player sitting on the bench can still benefit from pulling out the general principles and making some mental calculations before getting on the ice.

A player can make an assessment of probabilities from video footage of the goalie of the opposing team or from watching from the bench and represent them on a tree diagram as follows:



If the estimated probabilities are as in the diagram above, then the player's best strategy can be summarized as follows: "If the goalie is out of the goal the player should use a fake or a decoy and if he is deep in the crease, the player should shoot"

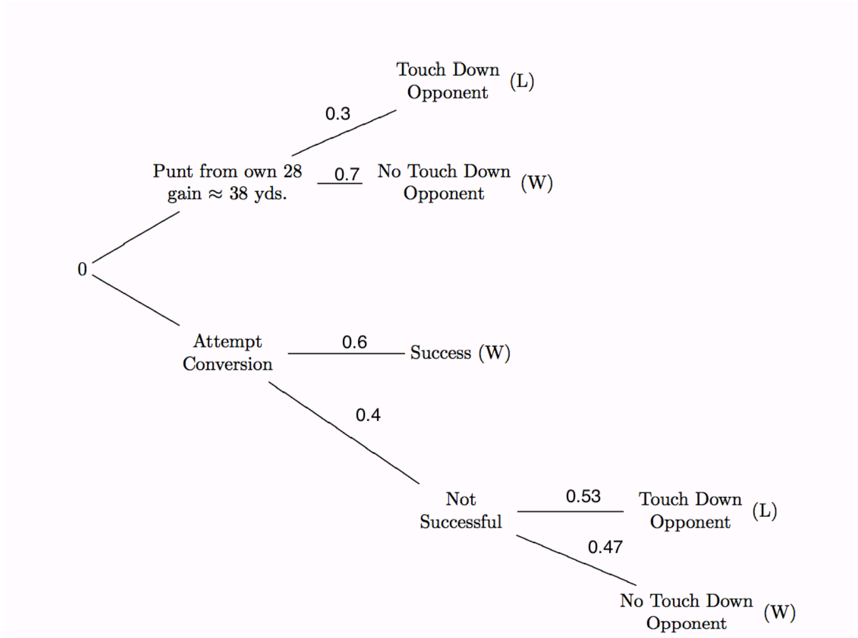
(a) Suppose on the other hand the probabilities are as in the following diagram, what is the best strategy for the player?



Note If the goalie is more dynamic and can switch position quickly perhaps sometimes coming out of the crease as the shooter approaches or moving back into the crease, the player should use the analysis for a simultaneous move game given in the subsequent material.

1.4. **Extra Example: Fourth Down, Go for it or kick it?** In American Football, when a team gains possession of the ball, they have 4 chances to advance at least 10 yards. If the team does not advance at least 10 yards, they lose the ball after the fourth attempt. Instead of making a fourth attempt, many coaches either punt the ball down the field or kick for a field goal. Each play is called a "down" and the strategy of punting or kicking the ball on fourth down was challenged by David Roemer [1] in his paper `Do Firms Maximize? Evidence from Professional Football.` It seems that coaches are hesitant to go for it on fourth down (attempt a fourth down conversion) because of the criticism it draws. For example, `such a play by Bill Belichick` on Nov. 15, 2009 drew much criticism when it resulted in failure and the opposing team (The Colts) won the game with a touchdown.

Brian Burke of AdvancedFootballStats.com has developed a `calculator` to calculate the winning probability for the team with the ball for any given state in the game. In this `instance`, we can see his estimated probabilities and a breakdown of his calculation of the probability of a win for the Patriots on the tree diagram below:



With the score at 34-28 in favor of the Patriots and two minutes left in the game, the Patriots attempted to gain the two yards necessary to get their fourth down conversion at their own 28 yard line. According to statistical evidence, Burke claims that a punt from this position gains 38 yards on average which would have placed the Colts on their own 34 yard line giving them about a 30% chance of making a touchdown. On the other hand the probability of success for a fourth down conversion was about 60% and if it failed, the chances that the opposing team would score a touchdown (from statistical evidence) from that position was about 53%. The chances of getting a field goal from their own 28 yard line was 0. With two minutes left in the game, the Colts had only one timeout left so a successful conversion would have won the game for the Patriots and a touchdown would have (and did) won the game for the Colts.

(a) Using the above diagram, calculate the probability (in the given situation) of a win for the offense using the strategy of punting the ball.

(b) Using the above diagram, calculate the probability (in the given situation) of a win for the offense using the strategy of attempting a fourth down conversion.

Obviously using these assessments of probability, the better strategy(for this situation) was to attempt the conversion because it offers the greatest probability of winning the game and the strategy will gain more wins for the team in the long run. However, as we have noted before, **the better long term strategy does not always result in a win** and this leaves the door open for criticism.

References

1. David Roemer, *Do firms maximize? evidence from professional football*, Journal of Political Economy **114** (2006), no. 2, 340–365.